**Homework 5, due 10/29**

1. Let $f : D \to D$ be holomorphic on the unit disk $D$, such that $f(0) = 0$.
   (a) Prove that
   $$|f(z) + f(-z)| \leq 2|z|^2$$
   for all $z \in D$.
   (b) Suppose that $|f(z_0) + f(-z_0)| = 2|z_0|^2$ for some $z_0 \neq 0$. Show that then $f(z) = e^{i\theta} z^2$ for some constant $\theta \in \mathbf{R}$, for all $z \in D$.
2. Let $A \subset \mathbf{C}$ denote the half-disk $A = \{z : |z| < 1, \operatorname{Re} z > 0\}$, and $B$ denote the quarter plane $B = \{z : \operatorname{Re} z, \operatorname{Im} z > 0\}$.
   (a) Find a biholomorphism $f : A \to B$.
   (b) Find a biholomorphism $g : B \to D(0, 1)$ to the unit disk.
3. Does there exist a surjective holomorphic map from the unit disk to $\mathbf{C}$?
4. Show that the map
   $$z \mapsto \int_1^z \frac{dw}{(1 - w^n)^{2/n}}$$
   is a biholomorphism from the unit disk to the interior of a regular $n$-gon.